

Problematizing Students' Thinking Discussion Protocol

Group #	M2
Phase 1 will be led by:	Catherine Brizard
Phase 2 will be led by:	Stefania Iannantuoni
Intended audience (i.e., who will be your students in the activity you enact):	Cycle 2 - Grade 3
Time needed for the activity:	45-60 minutes

Instructional Goals

a. Mathematical Content Goals

- Through this activity, students will...
 - Students will develop an understanding that units must be **identical**. They must be the same in order for an iteration to yield a single measure.
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 - Students will develop an understanding that units must **tile**. In this lesson, in particular, students will learn that they are translated “end-to-end” with no gaps.
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 - Students will develop an understanding that units are **iterated**. A unit is repeatedly applied to obtain a measure. This is the idea that we can re-use units. For example, a measure of length is obtained by moving (translating) a unit-distance a finite number of times.
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 - Students will develop an understanding that units can be partitioned into **partial units** (fractional units). Partial units allow for more precise measures.

b. Mathematical Practice Goals

- Students will learn to explain what they did when solving a mathematical problem.
- Students will learn how to justify their thinking (e.g., explaining why something is true).

Expectations for Students' Participation in Mathematical Activity

- In this activity, we expect students to...
 - **Explain their thinking**. This is important for encouraging students to explain their strategies. More generally, explanation is a key mathematical practice.
 - We expect students to **listen to each other**. This is important to ensure that students learn about different ways to compose the numbers.
 - We expect students to **be prepared to try to make sense of each other's ideas** in order to learn new ideas.

Materials:

- Books that are all the same length
- Bookshelf
- Whiteboard
- Two different coloured pens to write on the whiteboard

Focal Questions/Actions to Problematize Student Thinking

Mathematical Idea	Action to Problematize Student Thinking	Question to Problematize Student Thinking	Why is this Question/Action Important for Problematizing Student Thinking?
Identical Units	Introduce the idea of measuring by taking steps (counting as you step) while walking past the bookshelf. Ask the students if this will measure the bookshelf. Take uneven steps. Count how many steps you take.	“Okay I’m going to try to measure the bookshelf with my steps. Look carefully at what I am about to do. I will ask one of you to describe what I did...How did I measure the bookshelf? Who can describe what they noticed about what I just did?... Can we use my method to measure the bookshelf?”	This action and these questions are important for problematizing students’ thinking as it helps them understand the concept that units must remain identical when measuring. Through these series of questions, it promotes students’ thinking by observing what has taken place and discuss their various ideas in terms of measurement.
Tiling	Lay the identical books out one by one, leaving spaces between some of the books (measure all the way to the end).	“So, we agreed that we can use the books to measure the bookshelf because they are all the same length. Now I’m going to use the books to measure. Will this measure the bookshelf?...Why/Why not?”	Depending on the way in which the bookshelf is measured, it will allow students to gain an understanding of the concept of tiling. By showing students different ways of tiling (proper tiling, tiling error) and prompting their thinking it will allow them to determine the appropriate length of the bookshelf, as well as how they have attained their answer.
Iteration	After the students finish measuring, they should not reach the end.	<i>Ask others to comment on this:</i> “So far, we have used ____ books. Does that mean that the bookshelf is ____ books long?...Why not?”	Iteration is an important process in problematizing student thinking as students should be encouraged to repeat the process of measurement and discover new solutions.
Fractional Units	After finishing measuring, there should be a small space at the end (at least a $\frac{1}{4}$ or $\frac{1}{2}$ book).	“What do you notice about our books? So far, we have used ____ books. Does that mean that the bookshelf is ____ books long?” <i>If they don’t come up with the space at the end of the books, press on their thinking for what to do with this extra space. Say something like...</i> “What about this space at the end? Do we need to worry about this part? Why?”	Leaving space at the end of the measurement is important for problematizing students’ thinking as it permits students to gain insight into fractional units. Asking students what they notice about the measurement can encourage them to express different thoughts and ideas. Students may not grasp the concept of fractional units; therefore, it is important to press on their thinking to find out what we could do with the extra space.

Anticipated students thinking (e.g., how will they respond to the questions/actions and why):

Idea of Identical Units:

- “[That is not okay because] she took some small steps and some big steps”
- *When asked how to fix the problem: “keep your feet together.”*
- *When asked how to fix the problem: “you can do all long steps.”*
- *The teacher probed students’ thinking further by asking a student to show how they could fix what she had down. The student walked heel to toe and the teacher asked someone else to explain “Why was her method different from mine?” Students responded with:*
 - “She took small steps and they were all the same.”
 - “because when she did it she always did the same length between each foot, and you did some big and some small.”
- *When the teacher probed students’ thinking further by laying textbooks so that some lay lengthwise and others widthwise, some students were not sure if that would work. They responded with:*
 - “because it could or couldn’t because this length (pointing to one side) could be the same as this length (pointing to adjacent side)”
 - “because I’m not sure if the length of this one sideways is the same as the other way. Like the bottom is the same as the side length.”

Idea of Tiling:

- “[No] because for one you need more books and you have to put them like that (putting the books side by side)”

Idea of Iteration:

- “[No] we need more books.”
- *When asked if there is a way to measure without extra books: “You can count all these (meaning the books already laid down) and then move that book, these books to the end”*
- *When asked if there is a way to measure without extra books: “I was thinking that you should keep this there, move this one there, than move this one there, than you move the last one there” (while saying this, he first took the first book and moved it underneath the fourth book, than he took the second book and moved it underneath the new fourth book and then took the third book and moved it underneath the new fourth book).*
- *When asked if there is a way to measure without extra books a student proposed opening each book so that they would be longer.*

Different methods students might propose for iterating:

- Use one book, pick it up, mark the place with your finger and put it down.
- Use one book, flip it over and continue doing so until reaching the end.

Idea of Fractional Units:

- “Um no because the book is too big to measure this space”.
- “[No] because it is not exactly one book”

As seen above, students will respond in various ways. We anticipate they will respond in these ways based on the questions by which they are prompted by. Depending on the questions we ask students, it will likely influence the way in which they respond. We will ask different questions in order to press student thinking and encourage their critical thinking process.

Ways we will adjust the activity if it is too challenging:Provoking the idea of Identical Units:

- For this idea, we will start by taking proper steps; from the beginning of the bookshelf to the end of the bookshelf. We will put one foot in front of the other but ensuring that the tip of one shoe touches the back of my other shoe. Walking along the bookshelf, while putting one foot in front of the other, we will illustrate how we want students to measure the bookshelf.
- We will then have students measure twice, once with gaps and once without and see what the measure is. Thus, we will say, "Okay, I'm going to measure again. What do you notice that I did differently this time?"

Provoking the idea of Tiling:

- They'll see that they get two different measures. Then we will discuss whether it is okay to have two different measures. We will have them "investigate" with a partner at their desk by measuring the length of their desk using markers or paperclips. Then they could come back together to share their ideas and relate back to the bookshelf. If they are still struggling, we will relate to their own heights. We will ask: "can you have two different heights?"

Provoking the idea of Iterating:

- For this idea, we will orient students' thinking to one another by asking: "how many books have we used? So, how many books did it take us to see how long the bookshelf is?"
- In addition, you can have them predict how many more they think they will need to measure the bookshelf. When explaining, students will come up and gesture where they "see the books". You can note that they are using their hands as if it is a book to count - could we do that using one of the books we already used? If they are still struggling, we will ask, "What if I moved this book here - could we do that to try to figure out how many books long the bookshelf is?"

Provoking the idea of Fractional Units:

- For this idea, we will separate the books that are placed at the end and do this in front of all the students to see. We will then say something like: "As you can see, there are spaces in between the books at the end. Do we need to worry about this part? Why?"
- "Earlier, you said that it was not okay to have this space. Is it okay to have this space at the end? Why or why not?"

Ways we will adjust the activity if it is too easy:Provoking the idea of Identical Units:

- For this idea, rather than showing the students to take steps and measure their footsteps (count), we will indicate to the students that we want them to simply use their feet to measure the bookshelf. We will say something like: "Using both your feet, measure the bookshelf." This phrase will be the only indication we will provide the students. Based on our instruction, we will observe how students perform this task.

Provoking the idea of Tiling:

- For this idea, we will now show the students the books laid out in front of them and say something like: "Ok now that we measured the bookshelf with our feet, using ONLY these books (show books), measure how long the bookshelf is." This phrase will be the only indication we will provide the students. Based on our instruction, we will observe how students perform this task.

Provoking the idea of Iterating:

- For this idea, we will ask students to explain their thinking in detail and provide an explanation for how they measured the length of the bookshelf, while indicating the number of objects needed to find the total length.

Provoking the idea of Fractional Units:

- For this idea, we will separate the books at the end and do this in front of all the students to see. We will then say something like:
 - “What did we just do?”
 - “Why did we do this?”
 - “What does this mean?”
 - “Do we need to worry about this part? Why or why not?”

Activity Sequence

Phase 1
Instructional goals that will be targeted during this part of Phase 1: ● Students will develop an understanding that units must be identical. They must be the same in order for an iteration to yield a single measure. ● Students will develop an understanding that units must tile. In this lesson, in particular, students will learn that they are translated “end-to-end” with no gaps. ● Through this activity, the students may also develop some initial ideas about three important ideas about measurement: ○ There are different ways/attributes to measure (length, area, volume, angles, mass) ○ We have tools to measure (rulers, protractors, scales) ○ We use units to measure (cm, cm², liters, kg)
Introduce your group and establish expectations for student participation. Discuss your general expectations for student participation. Say something like... Before we get started, we want to discuss how we are going to participate. We have several expectations for how we are going to participate. Our first expectation is that it is really important is that everyone should be ready to explain how you are thinking. Our focus will be on explaining different strategies and understanding why. Pause and ask someone to restate this expectation or explain the expectation. You might choose to write the expectation or use pictures to help students follow (e.g., a mouth to show explaining). Second, it is really important that everyone listen to one another. Why is it important to listen to each other? Pause and ask someone to restate this expectation or explain the expectation. You might choose to write the expectation or use pictures to help students follow (e.g., an ear to show listening). Finally, we also expect that everyone be ready to explain or build upon how another student is thinking. Sometimes we might ask you to say in your own words what another student said so that we can learn from each other. Pause and ask someone to restate this expectation or explain the expectation. You might choose to write the expectation or use pictures to help students follow. You might check in with students to see if they recall the expectations. Say something like... “Let’s review what are expectations are. Who would be willing to share with us one of our expectations?” [Continue reviewing the expectations]. We’re also going to be using something called a silent thumb today. Whenever one of us asks you to think about a problem, we’re going to ask you to show a thumbs up, like this (demonstrate), to let us know that you are ready with your answer and with an explanation for how you got your answer. HOOK: “Hey everyone, we ran into a problem with my bookshelf, and we were wondering if you could help me figure it out. We need to know how many books we can fit on one shelf. What do we need to do in order to find out how many books we can place there?” If student Says “we need to measure” I would say “ Ok, let’s talk about that idea.”

Elicit Students' Initial Ideas about the Focal Concept (e.g., Measurement)

Here you will want to elicit students' initial ideas about the focal concept (e.g., measurement). Do not worry about coming to consensus about all these ideas. Through the following discussion, students will elaborate on these ideas. Plan out what questions you will ask here to elicit their thinking.

We will ask:

- “What is measurement?”
- “Why do we measure?”
- “What are you doing when you measure?”
- “Are there different methods of measuring?”

Allow a few moments (after asking each question) for students to consider how they know their answer is correct. We might allow a few moments for students to talk together in pairs to discuss their ideas.

After asking each question, we will remind students to show us their silent thumb. Give students a few moments to consider the questions asked. If not, everyone has their silent thumb up, we will ask the question one more time. If need be, we will do this for each question asked.

Record students' ideas on the whiteboard.

Introduce the Activity/Problem

Here, you will want to introduce the problem that the students will be working on together (e.g., measuring the bookshelf). Consider how you will introduce the activity to the students and how you might elicit their initial ideas about the problem.

Introduce the idea of measuring by taking steps (counting as you step) while walking past the bookshelf. Ask the students if this method will measure the bookshelf. Take uneven steps. Count how many steps you take.

“Okay I’m going to try to measure the bookshelf with my steps. Look carefully at what I am about to do. I will ask one of you to describe what I did.

Provoke the Idea of Identical Units

Elicit students' observations and explanations.

- “Can someone tell me what I did?”
- “How did I measure the bookshelf?”
- “Who can describe what they noticed about what I just did?”
- “Can we use my method to measure the bookshelf?”

Elicit students' explanations.

- “Who thinks we can use my method of measuring? Show me your silent thumb.”
- “Show me your silent thumb if you disagree with my method of measuring.”
- “Why did I measure the bookshelf this way?”
- “What other methods can we use to measure the bookshelf?”

Elicit students' ideas of how to correct what you did.

- “What could I do differently?”
- “Was this a good way to measure?”
- “Let’s try other ways of measuring”

Try out what the students suggest.

Was that a good way to measure?
“What do you think of what ____ suggested?”
Why was that a good way to measure?

Record students’ ideas on the whiteboard.

Provoke the Idea of Tiling

Elicit students’ observations and explanations.

- “Can someone tell me what I did?”
- “How did I measure the bookshelf?”
- “Who can describe what they noticed about what I just did?”
- “Is this a good way to measure?” [leaving space]

Elicit students’ explanations.

- “Who thinks we can use my method of measuring? Show me your silent thumb.”
- “Show me your silent thumb if you disagree with what was said.”
- “Why did I measure the bookshelf that way?”
- “What other methods can we use to measure the bookshelf?”

Elicit students’ ideas of how to correct what you did.

- “Is there something I do differently?”
- “Was this a good way to measure? Why or why not?”
- “Let’s try other ways of measuring. What is another way to measure?”

Try out what the students suggest.

Was that a good way to measure?
“What do you think of what ____ suggested?”
Why was that a good way to measure?

Record students’ ideas on the whiteboard.

Transition – Put one book down. Ask the students to tell me how to place the second book down. Then ask “Why did ____ suggest I put it so that it touches the other book?”

Phase 2

Instructional goals that will be targeted during Phase 2:

- Students will develop an understanding that units are **iterated**. A unit is repeatedly applied to obtain a measure. This is the idea that we can re-use units. For example, a measure of length is obtained by moving (translating) a unit-distance a finite number of times.
- Students will develop an understanding that units can be partitioned into **partial units** (fractional units). Partial units allow for more precise measures.

Remind students of your group's expectations for participation.

(Commend students on what they were doing well when working together). Remember, during our discussion, to be prepared to explain your thinking and listen carefully and be ready to build off one another's contributions. (If you introduced any language earlier, such as "restating," be sure to remind them of that language here).

Provoke the Idea of Iteration

Elicit students' observations and explanations.

- "The books are laid out on the bookshelf but it seems like there's not enough books to fill up this bookshelf. So, what do we do?"
- "How many books have we used?"
- "Does that mean that the bookshelf is ____ books long? Why?"

After asking these question, we will remind students to show us their silent thumb. Give students a few moments to consider the questions asked. If not everyone has their silent thumb up, we will ask the questions one more time.

Elicit students' explanations.

Now we want to know what students notice about the total length of the amount of books needed to measure the bookshelf. This is where a lot of the mathematical ideas will emerge for us to ask questions about and build upon.

It is important to represent the students' mathematical thinking on the whiteboard.

We will be sure to plan ahead regarding how we will represent students' thinking in order to highlight important mathematical ideas related to your goals.

We will ask:

- "Show me your silent thumb if you have an answer in mind."
- "Who would be willing to share how many books they noticed?" – We will start by asking for students who think they cannot measure. Have him or her explain their explanations. Then we will ask for students who think they can measure and see their explanations.
- "How did you find that answer?"

List all the answers. If more than one answer exists, say: "It looks like we have several ideas. As we share, let's see which of these ideas we need to revise."

Elicit students' ideas of how to correct what you did.

- "What could I do differently?": Here, I will orient back to the students who thought they couldn't measure because "we didn't have enough books". I will have students get up and demonstrate the method that another student proposed.
- "Are there different methods of measuring?"
- "What if I only used 1 of these books to measure the whole bookshelf... would my answer still be the same?"

- “If I were to open each book, we our answer still be the same? Why or why Not?”
- “Was that a good way to measure? Why or why not was this a good way to measure?”

Record students’ ideas on the whiteboard.

Provoke the Idea of Fractional Units

Elicit students’ observations and explanations.

- “Can someone tell me what I did?”
- “How did I measure the bookshelf?”
- “What do you notice about our books?” (Are they the same size?) – We will start by asking for students who think they cannot measure. Have him or her explain their explanations. Then we will ask for students who think they can measure and see their explanations.

List all the answers. If more than one answer exists, say: “It looks like we have several ideas. As we share, let’s see which of these ideas we need to revise.”

Elicit students’ explanations.

- Similarly to above: we will start by asking for students who think they do not need to measure the extra space. Get out their explanations. Then we will ask for students who think we need to include that space in our measurement. Come back to the students who thought it was not important and ask them to explain what the other student is saying: “Why do they think that we need to include that part?”

If they don’t come up with the space at the end of the books, press on their thinking for what to do with this extra space. Say something like... “What about this space at the end? Do we need to worry about this part? Why or why not?”

Elicit students’ ideas of how to correct what you did.

- “How long do you think that _____ space is?” – A student might bring up the idea of a specific unit (i.e. cm/m/cm²/mm). If a student brings up this notion, I will bring back the idea that for this activity we are only using books as measurements.
- So I will ask: “How many books do we need to measure the bookshelf?”

Record students’ ideas on the whiteboard.

Conclude the activity

- Say something like...
 - “Nice work working together. We noticed that students found many different ways for measuring the bookshelf. What were some of the different ways to measure the bookshelf?” [elicit ideas from the students].
 - We will then have students go back to their desk and draw or write all the ways they were able to measure the bookshelf. Once they have done this, we will have them share what they drew or wrote in a group discussion.
 - To conclude the activity, we will comment on how the group participated in the mathematical activity, given the expectations we laid out. For example, we especially liked how everyone _____ (contributed to our whole class discussion). We will be as specific as possible about what we noticed the children were doing well throughout the enactment.